ERRORS IN S_{11} MEASUREMENTS DUE TO RESIDUAL SWR OF THE MEASURING EQUIPMENT

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Abstract

Errors in measurement of S_{11} due to residual SWR of the slotted line or directional coupler are calculated using admittance considerations. This more complete treatment produces, for some practical conditions, major differences from prior references for phase errors.

Introduction

Although slotted lines have been used for measuring impedance for over three decades and directional couplers have been used for at least one decade, the phase errors due to the residual SWR and the finite directivity of these devices has not been correctly calculated. The mathematics of the phase error, for the general case, has so far proven to be intractable. Results are presented here of a computer program in which these errors were calculated for a wide range of parameters using numerical methods. These errors are compared to the incorrect relationships that have been used by others. The errors can be very closely described by the equation

$$\Delta\phi = \left(1 + 3|\Gamma_x| - \frac{1}{4}\sin\pi|\Gamma_x|\right)\sin^{-1}\left|\frac{\Gamma_m}{\Gamma_x}\right| \qquad (1)$$

in which Γ_x is the reflection coefficient of the unknown (S_{11}) and Γ_m is the reflection coefficient of the intervening mismatch when a perfect load is placed behind it (residual SWR).

The phase errors given by the manufacturers of precision phase measurement equipment are given in Table I.

TABLE I

ERROR	Manufacturer
$\pm\left[A + \tan^{-1}\frac{B}{\lvert\Gamma_x\rvert} + C\lvert\Gamma_x\rvert\right]$	
$\pm\left[2^\circ + \tan^{-1}\frac{.01}{\lvert\Gamma_x\rvert} + 3\lvert\Gamma_x\rvert\right]$	Rantec
$\pm\left[2^\circ + \tan^{-1}\frac{.01}{\lvert\Gamma_x\rvert} + 3.5\lvert\Gamma_x\rvert\right]$	Wiltron
$\pm\left[0.5^\circ + \tan^{-1}\frac{.003}{\lvert\Gamma_x\rvert} + 4\tan^{-1}(.015\lvert\Gamma_x\rvert)\right]$	Hewlett-Packard (computer-corrected)
$\pm\left[0.25 + \tan^{-1}\frac{.0015}{\lvert\Gamma_x\rvert} + 4\tan^{-1}(.005\lvert\Gamma_x\rvert)\right]$	Hewlett-Packard (computer-corrected with phase lock)

Error A in table I is contributed mostly by the low-frequency information processing circuits. Error B is contributed by the finite directivity of the directional coupler used in picking up the reflected wave. Directivity η is given by

$$\eta = 20 \log B$$

B = .01 indicates 40 dB directivity

Error of the term containing C is contributed by residual output SWR. This error was originally taken as $\tan^{-1}|\Gamma_m||\Gamma_x| \approx 60^\circ|\Gamma_m||\Gamma_x|$. A residual output SWR of 1.1 would give $|\Gamma_m| = .05$ which would give C = 3. Lately Hewlett-Packard has begun to use $4\tan^{-1}|\Gamma_m||\Gamma_x|$.

The equations for phase errors are in error in two respects: first, the equations for residual SWR are not correct, and second, directional coupler directivity errors must be combined with residual SWR effects before equation 1 is applied. The manufacturers have been careful in not publishing explanations of the errors they have specified. The equations they give may fit their observed errors but the error mechanism is not correctly associated with the error. In fact an output SWR of 1.2 ($|\Gamma_m| = .091$) is common in specifications which for $|\Gamma_x| = 1$ would give an error of ±20.8° using equation 1, but according to the table, the error for the noncomputer-corrected measurement would be about ±6°.

Theory

The residual SWR may be the summation of smaller SWR's contributed by connector irregularities or transmission line imperfections. Even though these smaller contributors may be many and in different electrical planes they can all be tuned out by one tuning probe having two degrees of freedom: susceptance and electrical plane. Then the total residual SWR can be represented by one susceptance in any electrical plane. The plane of this discontinuity is not limited to the plane of the connector or to the plane of the unknown, but it may be in any plane. The experimental arrangement for a slotted line is shown in Figure 1. The term Γ_x is the intervening mismatch (causing the residual SWR), Γ_r is the resulting reflection coefficient that will be measured on the slotted line, and ℓ is the distance between the unknown and the lumped discontinuity causing the residual SWR.

The manner in which the discontinuity causes phase error is shown in Figure 2. A $SWR_m = 1.5$ is used in conjunction with a short circuit giving a normalized admittance of $Y_x = \infty + j\infty$. The mismatch gives an equivalent susceptance B_m of $B_x = .409$. The Smith Chart of Figure 2 shows admittance adding in the plane of the discontinuity B_m for various values of ℓ/λ, the electrical distance to the discontinuity. The greatest phase shift occurs when the discontinuity susceptance takes the total admittance from $0 - jB_m/2$ to $0 + jB_m/2$ and the phase shift is 46°.

The admittance adding process for $|\Gamma_x| = 1.5$ and $SWR_m \leq 1.5$ is shown in Figure 3. In order to simplify the calculation, the observer moves toward the generator from the unknown until the normalized admittance of the unknown is real and greater than one (Y_x = 3.0 + j o for Figure 3). When ℓ/λ is measured from this position, the process used in figure 3 is similar to that used in Figure 2. As ℓ/λ is increased, the circle labeled Y_x is produced which is the locus of the admittances of the unknown. When the susceptance of the discontinuity is added, the circle $Y_x + jB_m$ is generated. The maximum phase shift is again nearest $0 \pm j0$ and almost centered about the real axis. Figure 3 also shows what happens to the magnitude of reflection coefficient. The resulting SWR, SWR_R is given by

$$SWR_R = (SWR_x)(SWR_m)^{\pm 1} \qquad (2)$$

the + sign giving the maximum and the - sign the minimum. Equation 2 is in agreement with Altman[2] who gives

$$|\Gamma_r| = \frac{|\Gamma_m| \pm |\Gamma_x|}{1 \pm |\Gamma_m||\Gamma_x|}$$

But the phase shift is in agreement with no one.

Numerical Method

Many attempts to derive an analytical expression for the phase error have led to intractable arrays of algebra. Therefore a numerical method was set up on the computer doing the same operations as were performed in Figure 3. Maximum phase error occurs for the phase of the unknown reflection coefficient ϕ_x when $0^\circ < \phi_x < 90^\circ$. The computation is performed as follows:

1. With $\phi_{x1} = 0^\circ$ then Y_{x1} is calculated from $|\Gamma_x|e^{j\phi_{x1}}$
2. B_m is added giving a new admittance Y'_x and Γ'_x
3. ϕ'_{x1} is calculated from Γ'_x
4. $\Delta\phi_1 = \phi'_{x1} - \phi_{x1}$
5. ϕ_{x1} is incremented by 1° ($\phi_{x2} = \phi_{x1} + 1^\circ$)
6. Y_{x2} is calculated

   Repeat steps 2, 3 and 4 getting $\Delta\phi_2$
7. $\Delta\phi_2$ is compared to $\Delta\phi_1$

   If $\Delta\phi_{i+1} < \Delta\phi_i$ then the calculation is stopped and $\Delta\phi_i$ is displayed as the maximum error for the given $|\Gamma_x|$ and B_m.

If $\Delta\phi_{i+1} \geq \Delta\phi_i$ then steps 5 through 7 are repeated.

Using this method the phase error curves were calculated as shown in Figure 4. An equation has been found which fits these curves within a few percent which was given as equation 1.

Directional Couplers

The theory of the error from a directional coupler arranged to detect reflected power has the desired reflection coefficient of the unknown Γ_x adding to the equivalent reflection coefficient Γ_m due to finite directivity to give the resultant Γ_r. The addition of these vectors gives maximum phase error when the vectors are perpendicular (based on the formulas used in Table 1) giving phase error of

$$\Delta\phi = \tan^{-1}\left|\frac{\Gamma_m}{\Gamma_x}\right|$$

If the mechanism were correct the maximum phase error would be

$$\Delta\phi = \sin^{-1}\left|\frac{\Gamma_m}{\Gamma_x}\right| \qquad (3)$$

but even this is incorrect. It has been shown[3] that the finite directivity of a directional coupler can be tuned out with a tuner. The basic mechanisms causing the finite directivity are equivalent to discontinuities on the output. The way in which they contribute to errors is the same as for other discontinuities between a perfect measuring device and the unknown.

Comparison of Error Expressions

Figure 5 shows the computer generated error curves compared to other expressions of error. The computer generated curves for SWR_m of 1.5 and 1.05 are shown as solid curves. The dashed curves show the error that should occur if directional coupler voltage addition were correct (equation 3). It should be noted that simplification of equation 12 of reference 3 also leads to equation 3.

Schafer[4] also produced an expression for phase error (eq.31) which reduces to

$$\Delta\phi = \pm 2\sin^{-1}\left[\frac{|\Gamma_m|}{|\Gamma_x|} + |\Gamma_m \Gamma_x|\right] \qquad (4)$$

and is shown as the dotted curves in figure 5. Equation 5 of Phillips[5] also comes very close to equation 4.

Conclusion

The residual SWR contributions to error in S_{11} measurements are correctly calculated using equations 1 and 2. Where directional couplers are used for S_{11} measurements, their directivity should be converted to effective output SWR and combined with other residual SWR using equation 2. Then total errors should be calculated using equations 1 and 2.

References

1. Rantec and Wiltron figures are from their sales literature and operating manuals. Hewlett-Packard figures are from Adam, Stephen F., "A New Precision Automatic Microwave Measurement System", IEEE Transactions on Instrumentation and Measurement, Vol. IM-17, No. 4, December 1968, pages 308-313.
2. Altman, J.L. Microwave Circuits, D. VanNostrand Co., Inc., N.Y. N.Y, 1964, pages 393-395.
3. Schafer, G.E. and Beatty, R.W.,"Error Analysis of a Standard Microwave Phase Shifter," Journal of Research of the N.B.S., Vol. 64C, No.4, December 1960, pages 261-265.
4. Schafer, G.E., "Mismatch Errors in Microwave Phase Shift Measurement," IRE Transactions on Microwave Theory and Techniques, Vol. MTT-8, November 1960, pages 617-622.
5. Phillips, E.N., "The Uncertainties of Phase Measurement", Microwaves, February 1965, pages 14-21.

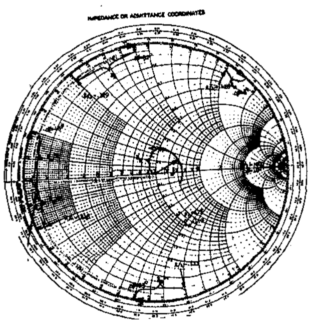

Figure 2 Phase Errors Possible with ρ_m=1.5 and $|\Gamma_x|$=1.0

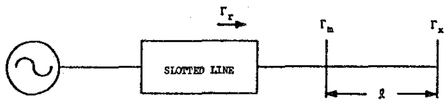

Figure 1 Microwave Circuit for S_{11} Measurements

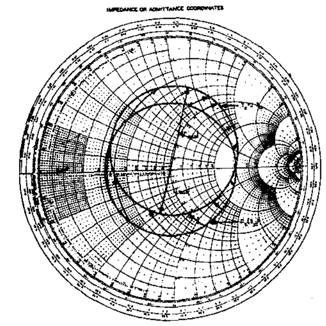

Figure 3 Range of Possible New Admittance Points for $|\Gamma_x|$=.5 and ρ_m=1.5 assuming B_m positive. Admittances are in the plane of Γ_m.

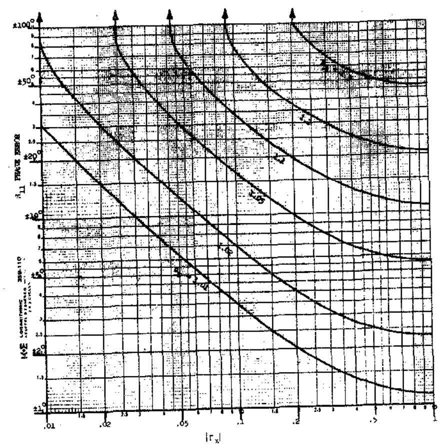

Figure 4 Computed Maximum Phase Errors in S_{11} for Various Values of $|\Gamma_x|$ and ρ_m.

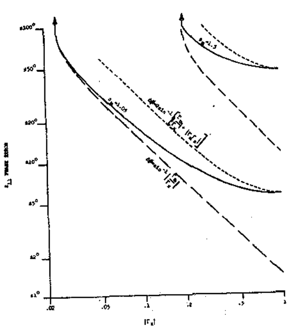

Figure 5 Comparison of Various Calculations of Phase Errors in S_{11} for ρ_m = 1.05 and 1.5.